The settling velocity of heavy particles in an aqueous near-isotropic turbulence

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The ensemble-average settling velocity, \( V_s \), of heavy tungsten and glass particles with different mean diameters in an aqueous near-isotropic turbulence that was generated by a pair of vertically oscillated grids in a water tank was measured using both particle tracking and particle image velocimetry. Emphasis is placed on the effect of the Stokes number, \( St \), a time ratio of particle response to the Kolmogorov scale of turbulence, to the particle settling rate defined as \( (V_s - V_t)/V_t \) where \( V_t \) is the particle terminal velocity in still fluid. It is found that even when the particle Reynolds number \( Re_p \) is as large as 25 at which \( V_t/u_k = 10 \) where \( u_k \) is the Kolmogorov velocity scale of turbulence, the mean settling rate is positive and reaches its maximum of about 7% when \( St \) is approaching to unity. In a direct numerical simulation of DNS results by Yang and Maxey (1993) and Yang and Lei (1998). This phenomenon becomes more and more pronounced as values of \( V_t/u_k \) decrease, for which DNS results reveal that the settling rate at \( V_t/u_k = 1 \) and \( Re_p < 1 \) can be as large as 50% when \( St \approx 1 \). However, the present result differs drastically with Monte Carlo simulations for heavy particles subjected to nonlinear drag (\( Re_p > 1 \)) in turbulence in which the settling rate was negative and decreases with increasing \( St \). Using the wavelet analysis, the fluid integral time (\( \tau_f \)), the Taylor microscale (\( \tau_\lambda \)), and two heavy particles’ characteristic times (\( \tau_{c1}, \tau_{c2} \)) are identified for the first time. For \( St < 1 \), \( \tau_{c1} < \tau_f \) and \( \tau_{c2} < \tau_\lambda \), whereas \( \tau_{c1} \approx \tau_f \) and \( \tau_{c2} \approx \tau_\lambda \) for \( St \approx 1 \). This may explain why the settling rate is a maximum near \( St \approx 1 \), because the particle motion is in phase with the fluid turbulent motion only when \( St \approx 1 \) where the relative slip velocities are smallest. These results may be relevant to sediment grains in rivers and aerosol particles in the atmosphere.


**I. INTRODUCTION**

The dispersion and the settling of heavy particles in turbulence are central in many natural and industry flows, such as for instance aerosol particles in the atmosphere, sediment grains in rivers, and the mixing of sprays in a combustor. The former (dispersion) problem has received most attention with a long line of publication on the dispersion of aerosol particles in homogeneous turbulence.\(^1\)-\(^5\) However, most of these dispersion experiments were carried out in a decaying homogeneous turbulence generated by wind and water tunnels.\(^1\)-\(^4\) On the other hand, very few experiments are available for the latter (settling) study. This is because these particles in wind and water tunnels would be brought away or influenced by the mean flow, making the determination of the particle settling velocity very difficult. Currently, only numerical investigations\(^6\)-\(^8\) are available for the study of heavy particle settling phenomena in homogeneous isotropic turbulence with no mean flow, but these numerical simulations cannot agree with each other. Whether the particle settling velocity would be increased or decreased by homogeneous turbulence without any mean flow still remains a question. This motivates us to study the problem experimentally and thus make the analogy between experimental and numerical approaches.

In a direct numerical simulation (DNS), Wang and Maxey\(^6\) suggested that in the linear Stokes drag range, the average settling velocity of heavy particles \( (V_s) \) in homogeneous isotropic turbulence could be as much as 40%-50% higher than the particle terminal velocity in still fluid \( (V_t) \) when the Stokes number was near unity. Here the Stokes number is defined as

\[
St = \frac{\tau_p}{\tau_k}
\]

a ratio of the particle response time to the Kolmogorov time of turbulence. Such an increase in the particle settling rate, \( (V_s - V_t)/V_t \), may be due to a particle preferential sweeping phenomenon for which these particles were accumulated near the low vorticity region of the flow.\(^6\) The maximum increase seemed to correspond to the strongest particle accumulation which occurred at \( St \approx 1 \). It revealed that the settling of heavy particles in homogeneous turbulence was influenced by small eddies of Kolmogorov scale. This view differs with the common understanding of the particle dispersion in which the processes are merely dominated by large-scale dynamics. Yang and Lei\(^7\) further investigated numerically on the role of turbulent scales to the aforemen-
tioned problem. They proposed that the Kolmogorov scale had little influence on the particle accumulation. In other words, the particle accumulation was mainly controlled by eddies that were generally one order greater than the Kolmogorov scale. Yang and Lei pointed out that the large eddy simulation (LES) is adequate for the settling study when the smallest resolved scale is smaller than \( t_w/2.5 \). \( t_w \) is the cut-off lengthscale corresponding to the maximum of the dissipation spectrum and is much smaller than the integral lengthscale. It should be noted that an increase of the settling rate, up to 50% when \( St \approx 1 \), was also possible in their numerical simulation.

Unlike the large enhancement found by DNS and LES studies, Mei suggested that turbulence had little influence on the particle settling rate in the linear Stokes drag range using Monte Carlo simulation, in which \( V_t \) was close to \( V_f \). Moreover, values of \( V_t \) may be smaller than that of \( V_f \) when the nonlinear drag law was considered in his Monte Carlo simulation. In other words, the effect of the nonlinear drag associated with turbulence may overwhelm the influence of the particle trajectory bias (preferential accumulation), resulting in a global decrease of the particle settling rate. Mei used a turbulence structure parameter \( r \) to demonstrate the effect of turbulence on the average settling velocity. Here

\[
 r = \frac{\omega_o}{(k_o u')} 
\]

where \( \omega_o \), \( k_o \), and \( u' \) were the typical frequency, wavenumber and root-mean-square velocity fluctuations of turbulence, respectively. Assuming that the Kolmogorov cascade could be applied to the turbulent flow field, the Kolmogorov time scale \( \tau_k = \tau_R e^{-1/2} \), where \( Re = \frac{u' L_i}{\nu} \), \( L_i \), and \( \nu \) are the integral time scale, the integral length scale, and the fluid kinematic viscosity, respectively. Thus, the parameter \( r \) may be rewritten as

\[
 r \approx 3.17 \times St^2, 
\]

where a Froude number, \( Fr = k_o u'^2/\nu = 1 \), and a particle turbulent Reynolds number \( Re_p = u' d_p/\nu = 4 \), were used. It is important to consider the actual range of \( r \), and/or \( St \) in both DNS and Monte Carlo simulations, because these two simulations predict different results of the particle settling rate due to turbulence. In Mei’s simulation, \( r \) varied from about zero to three which may correspond to 0.5<\( St <2.5 \) when \( Re_k = \frac{u' \tau}{\nu} \approx 30 \) where \( \tau \) is the Taylor micro length scale (\( \sim \sqrt{15u' \tau} \)). For the same \( Re_k \approx 30 \), values of \( St \) in DNS results varied from about 0.1–3. The DNS results not only found that values of the mean settling rate were all positive over the range of \( St \) at least from 0 to 3, but also showed that the largest increase of the settling rate occurred near \( St = 1 \) regardless of the linear and/or nonlinear drag laws were applied. On the contrary, Monte Carlo simulation predicted that \( V_s \approx V_t \) when the linear Stokes drag law was used and values of the settling rate were negative and decreased rapidly with increasing \( St \) when the nonlinear drag law was applied.

To our best knowledge, there were probably only two early (back to 1970’s) experiments, which measured the particle settling velocity in some turbulent oscillating water flows. A decrease of \( V_s < V_t \) was suggested from these oscillating water flows (Murray). Note that these oscillating flows were not homogeneous turbulence and the density ratio of particle to fluid \( p_f/\rho_f \) was only about 1.05 estimated from the Stokes law for the mean particle diameter \( d_p = 2 \) mm with \( V_t = 5 \) cm/s used in Ref. 11. Thus, a direct comparison between these oscillating water flow experiments and numerical simulations is not possible, because the latter assumes a flow field which is stationary and homogeneous with \( p_f/\rho_f = 1 \). Just recently, Lasheras and his co-workers investigated experimentally the effect of preferential concentration on the settling velocity of water droplets in decaying wind-tunnel grid turbulence. They found that once particle accumulation developed the mass loading locally also became significant and drove a downward flow, with the particles settling faster and moving collectively as a cluster. In other words, a significant increase of \( V_s > V_t \) was found for water droplets in decaying homogeneous air turbulence.

There is still a need for a well-designed experiment in order to resolve the discrepancy on the increase or decrease of particle settling rates due to stationary homogeneous turbulence. The well-designed experiment must provide wanted interactions between heavy descending particles and the turbulence which is stationary and homogeneous, over a range of \( St \) at least from 0 to 2 and with \( p_f/\rho_f = 1 \). The decaying homogeneous turbulence generated in water and wind tunnels may not be the best choice for this purpose, because there is a strong mean flow along the streamwise direction. The more appropriate turbulence is the one with statistically stationary characteristics and without mean flow velocities.

We have established such a turbulent flow field using a pair of vertically oscillated grids in a water tank, in which a stationary near-isotropic turbulence can be generated in the core region between the two grids, as verified by extensive LDV measurements. In addition, Villermas et al. used the same two-grid configuration to study the features of intense vortical structures in vibrating-grids turbulence (VGT) using a migrating bubble technique. Moreover, this two-grid near-isotropic turbulence generator combined with an aqueous autocatalytic reaction which can provide self-propagating chemical fronts has been applied to simulate turbulent burning velocities for premixed turbulent combustion. In this paper, we will report for the first time measurements of particle settling velocities in such a VGT flow. Heavy tungsten and glass particles with different mean diameters are used, so that the corresponding values of \( St \) and \( p_f/\rho_f \) can be ranging from 0 to 2 and up to 19.3, respectively. The average settling velocity \( V_s \) and the root-mean-square velocity fluctuations of heavy particles in both horizontal \( (u'_p) \) and vertical \( (w'_p) \) components as well as fluid velocity maps are measured using both the particle tracking velocimetry (PTV) and the particle image velocimetry (PIV). The following section reviews the experimental methods and the relevant parameter ranges used in the study, and is followed by a description of the wavelet transform technique which can investigate the evolutionary interactions between the fluid turbulence and the particles. Both are then applied to measure \( V_s, u'_p, w'_p \), and the temporal in-
formation of particle–turbulence interactions. These experimental results are compared with previous numerical results in attempt to address the aforementioned discrepancy. Finally, the limitations of the present experiment are discussed and areas for further study identified.

II. EXPERIMENTAL METHODS

A. VGT flow

Figure 1 shows schematically the VGT apparatus, the associated imaging acquisition arrangements for both PTV and PIV measurements, and three instantaneous images which reveal the evolution of a cloud of heavy particles descending in the VGT flow. A pair of specially designed grids were used to generate a stationary, near-isotropic turbulent flow field in the core region between the two grids (Fig. 1). There are two distinct flow regions in the VGT flow: Near-grid flow and near-isotropic regions. The latter has more than 12 turbulent integral lengthscales in height (~4 cm) in the case of a separation distance between the two grids H = 11 cm, a stroke S = 2 cm, a grid mesh size M = 3 cm, and an oscillating frequency f = 3–8 Hz. Thus, the corresponding grid turbulent Reynolds number, Reₚ = fSM/ν, is ranging from 1800 to 4800 in this study. A detailed treatment of this VGT flow can be found in Shy et al. For completeness, here we summarize briefly flow characteristics in the near-isotropic region. In it, mean velocities are essentially zero, turbulent intensities in all three directions are roughly equal with no more than 15% variations, and there is a ~5/3 energy decay slope at higher frequencies at least ranging from about 1–50 Hz (please see Fig. 8 in Ref. 13).

Table I lists some fluid turbulence properties, where u’ is the energy-weighted RMS turbulent intensity, ε is the energy dissipation rate, and vₓ, η, and τₑ are the Kolmogorov velocity, length and time scales, respectively. Here Reₜ is another turbulent Reynolds number based on u’ and the integral lengthscale Lᵢ. The common way to estimate Lᵢ from LDV data was to employ Taylor’s hypothesis of isotropic turbulence, in which Lᵢ = τₑu’. We used such a relation to estimate Lᵢ which was found to be a constant of ~3 mm for f varying from 3 to 8 Hz. The constant Lᵢ, independent of f, was because τₑ ~f⁻¹ and u’~f (see Ref. 13). Other authors also used the same relation to estimate Lᵢ for a stagnation flow and a fan-stirred turbulent flow. However, more experimental investigations on Lᵢ are still needed. It shall be noted that the viscous dissipation is commonly approximated by a relation of ε = A(u’³/Lᵢ), where A is a constant of order unity, see Hinze and Tennekes and Lumley, which states that viscous dissipation of energy may be estimated from the large-scale dynamics that do not involve viscosity. This inviscid estimation is one of the cornerstone assumptions of turbulence theory (first proposed by Taylor in 1935). It reveals that large eddies lose a significant fraction of their kinetic energy ½u’² within one eddy turnover time Lᵢ/ω. In this study, both u’ and Lᵢ were measured experimentally and then used to estimate the dissipation (ε) by assuming A = 1 in the above relation, see Table I for values of ε at different f. For instance, when f = 7 Hz, u’ ~ 1.68 cm/s, and Lᵢ ~ 0.3 cm, and thus ε ~ 15.8 cm²/s³. However, the true turbulent dissipation rate must be three dimensional involving nine components of the strain rate tensor, which could not be obtained directly using planar PIV measurements unless some assumptions are made. In order to measure directly the true turbulent kinetic energy dissipation rate in the VGT flow, full spatial PIV measurements are required and this is recently achieved. As can be seen from Fig. 4 of Ref. 22, the distribution of the dissipation rate field is a highly intermittent phenomenon, in which all high dissipations are concentrated in some small canonical structures, revealing the spotty features of fine scale turbulence. Therefore, the above estimate for the dissipation, ε ~ u’³/Lᵢ, may have some limitations, since the detailed relation between the fine scale intermittency and the viscous dissipation is still an unsettled problem in turbulence. For simplicity, in this work we employ the relation of ε ~ u’³/Lᵢ to estimate the dissipation and focus squarely on...
the settling velocity of heavy particles in the VGT flow using planar PTV and/or PIV measurements.

There is a cutoff grid-oscillation frequency for which \( f \) should not be greater than 9 Hz to avoid the unwanted circulating motion generated near the walls. This cut-off frequency limits the accessible range of \( \text{St} \) that can be conducted in the present experiment, as to be discussed later. In this study, the instantaneous fluid velocity field and the corresponding vorticity field of the VGT flow are also measured using the PIV method to further understand flow characteristics.

### B. Choice of particles and operating conditions

Finding the ideal experimental configuration is only half the challenge. It is very difficult to select the appropriate particles, discharge these particles uniformly into the VGT flow, and measure the corresponding particle settling velocity. In order to make the analogy with DNS results, the appropriate particles should have a density which is much greater than the fluid density, a mean particle diameter (\( d_p \)) which is smaller than or comparable to \( \eta \), and a terminal velocity (\( V_t \)) which is equal to or greater than \( v_k \). Therefore, the relevant parameter ranges could be closely matched with those assumed by numerical studies, in which \( \text{St}=0-2 \), \( Re_p<1 \) or \( Re_p^{-1} \), and \( V_t/v_k \approx 1-10 \), respectively. However, because of the experimental limitation (\( f<9 \) Hz), the accessible parameter ranges are limited. For instance, when the tungsten particles with \( d_p \approx 160 \) \( \mu \)m where \( \rho_p/\rho_f = 19.3 \) and the glass particles with \( d_p \approx 360 \) \( \mu \)m or 505 \( \mu \)m where \( \rho_p/\rho_f = 2.5 \) are used, the range of \( \text{St}=\tau_f/\tau_k \) can be varied from about 0.2 to 2, but the corresponding values of \( Re \) and \( V_t/v_k \) are greater than that of numerical studies, where \( Re_p = 17-39 \) and \( V_t/v_k \approx 10-30 \), respectively (see Table II). To reduce values of \( Re_p = V_t d_p / \nu \) and \( V_t/v_k \), smaller tungsten particles with \( d_p \approx 60 \) \( \mu \)m are used, at which \( Re_p \approx 1 \) and \( V_t/v_k \approx 5 \); however, the corresponding values of \( \text{St} \) are only limited from about 0.2 to 0.2. Though the present VGT experiment is unable to validate numerical data just at \( V_t/v_k \approx 1-30 \) where the enhancement phenomenon of the settling rate is most pronounced, the experimental results obtained at larger values of \( V_t/v_k \approx 5-30 \) are very interested in the sense that whether the settling rate would be increased or decreased by stationary homogeneous turbulence within any ranges of \( V_t/v_k \approx 1-30 \) has not been validated by experiments yet. In this study, both small (\( d_p \approx 60 \) \( \mu \)m) and large particles are employed, with an emphasis on the case of larger particles (\( Re_p > 1 \) and \( V_t/v_k = 5-30 \)), and thus whether the settling rate would increase by stationary homogeneous turbulence and be a maximum around \( \text{St} \approx 1 \) can be experimentally assessed for the first time.

One of the most difficult things in the present study is on how to discharge these heavy particles uniformly into the VGT flow, so that measurements of the particle settling velocity in near-isotropic region can be made. A number of particles releasing methods were tried, and the most satisfactory result came from a particle feeder which had two overlapping nets of small meshes with the same cross-sectional area of the tank. Before a run, the particles were carefully and uniformly brushed over the nets of small meshes of the particle feeder. Then the feeder was placed a few millimeters above the water surface, as shown in Fig. 1. A run began by vibrating the two grids to establish the wanted turbulence. After a few seconds, the particle feeder was then carefully moved down one or two millimeters below the water surface, such that a large cloud of heavy particles can be uniformly released from the top of the water tank. Note that if the feeder were moved down too suddenly or too slowly, the degree of uniformity of these descending particles will be reduced. In addition, the nets of small meshes must be placed slightly below the water surface, otherwise some particles may be suspended on the water surface due to the surface tension effect.

Figure 1 displays three instantaneous laser sheet images, with a field of view 3.5\( \times \)3.5 cm\(^2\) just in the near-isotropic region, which demonstrate the evolution of these descending particles in the measuring window. As a typical example using tungsten particles with \( d_p \approx 160 \) \( \mu \)m at \( f=6 \) Hz, Fig. 2 shows such an evolution of the effective particle number that is counted in the measuring window during a run. Here the effective particles are selected within a band of size and sufficiently illuminated after a thresholding of laser light. From Fig. 2, the effective particle number first increases up to a maximum and then remains nearly a constant of 340 for a period from about 0.85 to 1.18 s. Note that this period is selected to be the data sampling domain for measurements of
$V_s$ in this study. After 1.18 s, the effective particle number decreases. All experiments are carried out in a dilute two-phase flow, so that the particle–particle collisions can be neglected and the continuity equation for the carrier fluid may be remained unchanged by the addition of the particles.\textsuperscript{23}

**C. PTV and PIV measurements**

Measurements of the ensemble-average particle settling velocity in near isotropic region are via a high-speed laser sheet particle tracking velocimetry which is very similar to a video-based PTV technique used by Veber \textit{et al.}\textsuperscript{24} The light source was a Coherent 90-5W argon-ion laser, which was operated at 3.5 Watts of power. A laser sheet of about 5 cm in height and about 0.5 mm in thickness formed by a combination of cylindrical and spherical lenses cut the near-isotropic region between the two grids from the side of the tank (Fig. 1). The trajectory of these heavy settling particles in near-isotropic region were recorded using high-speed CCD cameras (Dalsa CA-D6 or Redlake MotionScope PCI 8000s) which were, respectively, operated at 262 or 500 frames/s with the resolution of 256×512 or 280×320 pixels. These images with 256 gray levels (8-bit) were stored in a data acquisition system. We processed only half of the original image with a field of view 3.5×3.5 cm$^2$ when the Dalsa camera was used. A computerized tracking procedure was used to determine the magnitude and direction of particle settling velocities. Specifically, we run a gray morphology operation to make the raw images more visible and remove unwanted noises,\textsuperscript{24} then determined each particle centroid in a given digitized image using an image processing software (OPTIMAS 6.5), and estimated the settling velocity of these particles which must be tracked over three consecutive images. Finally, we averaged all available data from each particle in a given digitized image to obtain an ensemble-average particle settling velocity ($V_s$). In order to measure particle settling velocities accurately using the PTV method, the particle number concentration has to be low. Moreover, the present particle feeder can only discharge as much as 450 particles in the measuring laser sheet domain (3.5×3.5 cm$^2$). These limitations prevent the possibility of observing the particle preferential accumulation phenomenon in this study.

As a typical example, Fig. 3 shows two instantaneous raw consecutive images and the corresponding velocity field image for tungsten particles at $f=$ 6 Hz where $St$=1, using the aforementioned PTV method. Even though the particle inertia is large in this case (Fig. 3) where $Re_p$=17, it is found that the present near-isotropic turbulence can increase the particle settling rate when $St$=1, as will be discussed in the next section. Using the same error analyses proposed by Veber \textit{et al.},\textsuperscript{24} the maximum error on the determination of $V_s$ due to the resolution error, the image processing error, and the particle tracking error is no more than 5%.

To obtain the root-mean-square velocity fluctuations of these heavy particles, we applied the digital particle image velocimetry which was essentially the same as that developed by Willert and Gharib.\textsuperscript{25} Using the cross-correlation technique, the displacement from the correlation data at the subpixel level is determined by a three-point estimator with the spline peak fit. The bias on the position of cross-correlation maximum may be as low as 0.02 pixel.\textsuperscript{25} The data processing is performed using the interrogation window which has a size of 32 by 32 pixels and is overlapped 75% with the next. In the case of fluid velocity measurements, the seeding particles were neutrally buoyant 15 $\mu$m polymer spheres, while glass and tungsten particles were used for particle settling velocity measurements. The seeding density was about 10–15 particles in a 32×32 pixels interrogated window. Thus, the RMS velocity fluctuations of these heavy particles in near-isotropic turbulence region can be obtained using the PIV technique.

**D. Wavelet transform of velocity signals**

Farge\textsuperscript{16} and her co-workers\textsuperscript{17} have used the method of wavelet transforms, as a new mathematical tool, to study statistically elementary structures of turbulence. In this study, we employ the same wavelet transform concept\textsuperscript{16} to investigate the temporal behaviors of heavy particles and fluid turbulence for the first time. From Farge,\textsuperscript{15} the wavelet transform $W(a,b)$ of a continuous real-valued time signal, such as for instance the fluid velocity signals (vertical direction) from the present PIV data $w(t)$, is defined as the inner product between $w(t)$ and an analyzing wavelet

$$W(a,b) = \int_{-\infty}^{\infty} w(t) \phi_{a,b}(t) dt,$$

(4)
where \( \phi_{a,b}(t) = (1/\sqrt{a})\phi((t-b)/a) \). Here \( a \) and \( b \) are the time dilatation and translation parameters, respectively, and the asterisk of \( \phi \) represents the complex conjugate. As pointed out by Farge, \(^{15}\) the analyzing wavelet \( \phi_{a,b}(t) \) is a dilated and translated version of a mother wavelet \( \phi(t) \) with resolution \( a^{-1} \) and position \( b \). Thus, the wavelet transform at specific positions on the time signal \( t = b \) and at a specific wavelet scale \( a \) which represents the period can be defined. The admissibility condition as used by Farge\(^{16}\) is applied on \( \phi(t) \)

\[
C_\phi = 2\pi \int_{-\infty}^{\infty} \frac{|\hat{\phi}(\omega)|^2}{\omega} d\omega < \infty, \tag{5}
\]

to assure that the inverse of the wavelet transform exists,
inverse of the frequency all exhibit a slope of nearly \( \frac{5}{3} \). At high frequencies, the spectra in the region of interest are much smaller than the instantaneous velocities from a series of time sequential PIV data planes. Therefore, the one-dimensional Eulerian frequency spectra can be estimated using the Fourier or wavelet transformation of these instantaneous velocities. At high frequencies, the spectra in the region of interest all exhibit a slope of nearly \(-\frac{5}{3}\). As a typical example, Fig. 5 reveals that the range of \(-\frac{5}{3}\) slope can expand from about 3 Hz to about 100 Hz, where the grid oscillating frequency \( f \) is fixed at 7 Hz.

\[ \text{Eq. (6)} \] we set \( \omega_c = \omega_u = 2\pi \) in this study. Hence, \( \phi_{a,b}(t) \) is centered at the position \( b \) and at a period \( a/2\pi(=f^{-1}, \text{the inverse of the frequency}) \) with the standard deviation \( \sqrt{\pi/2a^5} \).

III. RESULTS AND DISCUSSION

A. Fluid turbulence

After the two grids have vibrated to establish the wanted turbulence, PIV measurements in the region of interest were started \((t=0)\). Figures 4(a) and 4(b) display two instantaneous fluid velocity and vorticity maps of the VGT flow at \( t=0.3 \text{ s} \) and \( t=1.5 \text{ s} \), respectively, where the experimental conditions are \( f=7 \text{ Hz}, S=2 \text{ cm}, H=11 \text{ cm}, \text{the mesh size } M=3 \text{ cm}, \text{and } Re_g=4200. \) It is found that the magnitudes of these instantaneous velocity vectors are smaller than the energy-weighted turbulent intensity \((u')\) obtained from LDV measurements which is represented as a reference velocity arrow of 1.68 cm/s (see Fig. 4). The positive−negative values on vorticity maps indicate counterclockwise−clockwise motions. As can be seen from Figs. 4(a) and 4(b), the highest vorticities (positive: 7.13 s\(^{-1}\), the red color; negative: −5.4 s\(^{-1}\), the blue color) are very few. These vorticity distributions suggest that this VGT flow is lack of strong mean shear. We ensemble-average 20 instantaneous velocity maps selected from the sequential PIV data planes between 0.3 and 1.5 s to investigate the mean structure of the VGT flow. The results of the mean velocity field as well as its corresponding vorticity field are presented on Fig. 4(c). The average velocities at every point in the region of interest are much smaller than \( u' \), where the corresponding values of vorticities are also very small ranging from −2.72−2.65 s\(^{-1}\). This suggests that the VGT flow is a nearly zero-mean-shear turbulence. In addition, the one-dimensional Eulerian frequency spectra can be estimated using the Fourier or wavelet transform of these instantaneous velocities from a series of time sequential PIV data planes. At high frequencies, the spectra in the region of interest all exhibit a slope of nearly \(-\frac{5}{3}\). As a typical example, Fig. 5 reveals that the range of \(-\frac{5}{3}\) slope expands at least from about 3 to 100 Hz, indicating that the VGT flow is a true turbulent flow as has long been recognized by several researchers (e.g., Refs. 26 and 27). From the same velocity data, the corresponding values of skewness and flatness are approaching to zero and three, respectively. Thus, the present PIV data agree well with previous LDV measurements indicating that the VGT flow has some properties of isotropic turbulence.

B. Particle settling velocities and their uncertainties

It is important to test the sensitivity of the effective particle number to the mean particle settling velocity; typical uncertainties of \( V_s \) measurements. The experimental conditions are tungsten particles with \( d_p=160 \mu\text{m} \) at \( f=6 \text{ Hz} \), at which \( St=1 \) and \( V_s=10.31 \text{ cm/s} \).
At the same conditions for velocities, where the nonlinear drag law was considered. Where the experimental results are plotted, previous numerical results by Wang are found to be positive even for those particles with large values of \( R_e_p \approx 17–39 \) and \( V_i/V_k = 5–30 \). The corresponding increase of the settling rate reaches its maximum of about 7% for \( R_e_p = 25 \) at which \( V_i/V_k = 12 \) and about 4% for \( R_e_p = 39 \) at which \( V_i/V_k = 15 \), when \( St \) is approaching to unity. As \( St \) increases further, the settling rate drops and becomes negative when \( St > 1.5 \). These results reveal two points. First, as \( R_e_p \) increases, the enhancement of the settling rate diminishes. The second point confirms that the particle settling behavior is aware of small-scale turbulence, especially when the particle relaxation time is comparable to the Kolmogorov scale at which \( St = 1 \), as to be explained later using the wavelet analysis.

The effect of \( \rho_p/\rho_f \) on the settling rate can be partially discerned from Fig. 7. At about the same \( R_e_p (17–25) \), the increasing and decreasing trends of the settling rate with its peak occurred near \( St \approx 1 \) are essentially the same for both glass and tungsten particles whose density ratios are 2.5 and 19.3, respectively. This suggests that different values of \( \rho_p/\rho_f \) do not alter the trend that the maximum settling rate occurs at \( St \approx 1 \). However, the maximum settling rate tends to decrease slightly, as \( \rho_p/\rho_f \) increases. Because of the large difference in values of \( \rho_p/\rho_f \) between the present experiment \((\rho_p/\rho_f \approx 19.3)\) and previous DNS \((\rho_p/\rho_f \sim 1000)\) for the particle–air case, the comparison between two different systems needs to be viewed with some cautions. Nevertheless, the result of Fig. 7 does reveal one important point that the particle settling rate is maximum when \( St \approx 1 \), regardless of different values of \( \rho_p/\rho_f \) studied, either \( \rho_p/\rho_f \approx 20 \) or \( \rho_p/\rho_f \sim 1000 \).

As can be seen from previous numerical results as well as the present experimental data on Fig. 7, the relative change of the settling rate in turbulence may be characterized by two dimensionless parameters, \( St = \tau_p/\tau_k \) and \( V_i/V_k \). To better see the effect of \( V_i/V_k \), we plot the data regimes of the settling rate against \( V_i/V_k \) on Fig. 8 for both the present experiment and previous numerical predictions which are extracted from Fig. 16 of Yang and Lei, where the lower and upper boundary lines of these data regimes represent the settling rate data at \( St \approx 0.2 \) and at \( St \approx 1 \), respectively. For numerical predictions, the available ranges of \( V_i/V_k \) are from about 1 to 10, as represented by the dash lines (Regime I where \( R_e_k \approx 30 \) from DNS data) and by the dash–dot–dash lines (Regime II where \( R_e_k \approx 130 \) from LES data) with the shaded domain, respectively. For the present experiment, the available range of \( V_i/V_k \) is from about 5 to 30, as represented by thick solid lines with the shaded domain on Fig. 8. Using the simple power curve fitting on the aforementioned available data regimes, the anticipated data regimes for both the experiment where \( V_i/V_k \approx 1–5 \) and DNS data where \( V_i/V_k \approx 10–20 \) may be obtained, as also plotted on Fig. 8. It shall be noted that the general trends between these experimental and numerical data are very similar in the sense that values of the settling rate decrease with increasing \( V_i/V_k \) and

FIG. 7. Effect of the Stokes number on the settling rate and comparisons with numerical predictions including DNS and Monte Carlo simulations, where the nonlinear drag law was considered.
the influence of $St$ on $(V_i - V_f)/V_f$ diminishes as $V_i/v_k$ increases. At $V_i/v_k = 1$ and $Re_k \approx 30$, the anticipated experimental data reveal that the settling rate may vary from 16% to 38% when values of $St$ increase from 0.2 to 1.0. This anticipated experimental result is in very good agreement with Yang and Lei’s results in which they predict an increase of $(V_i - V_f)/V_f$ from 15% to 32% when $St$ varies from 0.2 to 1.0. However, for larger values of $V_i/v_k$, values of the settling rate for the present experiment are found to be much greater than that of numerical data for the same $Re_k \approx 30$. The reason for this quantitative difference is not clear, though we anticipate that the Taylor hypothesis and the settling phenomenon is still influenced by turbulence even when values of $V_i/v_k$ are as large as 20 based on the present experiment.

### C. Temporal evolution of fluid–particle interaction

Figures 9(a) and 9(b) display typical RMS velocity fluctuations of heavy particles in both transverse ($u'_p$) and vertical ($w'_p$) directions, respectively, as a function of the fluid RMS fluctuation ($u'$), using PIV measurements in the central uniform (near-isotropic) region. It is found that $u'_p$ is nearly uncorrelated with $u'$ and remains roughly a constant as $u'$ increases. On the other hand, $w'_p$ is correlated with $u'$. As $u'$ increases, values of $w'_p$ increase, for which $w'_p = 0.5u'$ for glass particles with $d_p \approx 360 \mu m$ and $d_p \approx 505 \mu m$, respectively. For tungsten particles of $d_p \approx 160 \mu m$, $w'_p$ is roughly uncorrelated or only weakly correlated to $u'$: for example, if we ignore just one data point at the smallest value of $u' \approx 0.7$ cm/s, then $w'_p \approx 0.2u'$. Because the other horizontal component of particle RMS velocity fluctuations, $u'_p$, is roughly equal to $u'_p$, a mean particle RMS turbulent intensity in all three directions may be defined as $\langle \sigma \rangle = (2\langle u'_p \rangle + \langle w'_p \rangle)/3$. Here $\langle \rangle$ represents an ensemble-average of all available data in the interesting region (the measuring window). Figure 9(c) shows the decreasing variation of $\langle \sigma \rangle/u'$ with increasing $St$. This result seems to support qualitatively previous DNS results where values of $\langle \sigma \rangle/u'$ may decrease from about 1 to 0.8 for the same range of $St$. Hence, a much more rapid decrease in values of $\langle \sigma \rangle/u'$ is found for the present study, probably because of the larger particle Reynolds numbers being investigated.

It has been known that the wavelet transform can detect coherent structures in turbulent flows as a form of apparently periodic structures in physical and wavenumber spaces or in frequency-time domains. In the present study, a continuous Morlet wavelet transform (previously discussed) is imposed on both the fluid vertical velocity fluctuations $V(t)$ and the particle settling velocity signals $V_p(t)$ to scrutinize the temporal behavior of particle–turbulence interactions for the first time. Thus, we may discern these characteristic time scales existing in both fluid turbulence and particle–turbulence interactions. By comparing these characteristic time scales in both cases, we may be able to explain why the settling rate at $St=1$ is a maximum.

Figures 10 and 11 display contour plots of wavelet transform of fluid velocity fluctuations $w(t)$ using neutral particles and particle settling velocity signals $V_p(t)$ for tungsten particles with $d_p \approx 160 \mu m$, respectively. Each includes two different grid frequencies: $f = 3$ Hz (a) and $f = 6$ Hz (b), respectively. In Figs. 10 and 11, the abscissa is the signal time $t$, the ordinate represents the characteristic period $(a/2\pi = \frac{1}{f_a^2})$, where $f_a$ is the characteristic frequency, and the color bar indicates the magnitude of the wavelet coefficient $W(a,b)$ [see Eq. (4)]. As can be seen from these figures, there are two characteristic periods that reveal obvi-
ously periodic oscillations for both $w(t)$ and $V_s(t)$ signals. For the case of fluid turbulence, two characteristic periods are, respectively, 0.4 s/0.2 s at $f = 3$ Hz and 0.21 s/0.11 s at $f = 6$ Hz. The first characteristic period corresponds to the integral time scale of fluid turbulence where $\tau_I \sim L/u' = 0.42$ s/0.21 s at $f = 3$ Hz/6 Hz, respectively. The second characteristic period is comparable to the Taylor microscale, because $\tau_L \sim L/u' = \sqrt{15}$ $\tau_L = 0.35$ s/0.12 s at $f = 3$ Hz/6 Hz, respectively. These two characteristic times for fluid turbulence, $\tau_I$ and $\tau_L$, are indicated as the dashed lines with arrows in Fig. 10. Similarly, the two characteristic times for particle–turbulence interactions ($\tau_{c1}, \tau_{c2}$) are shown in Fig. 11 for both $f = 3$ Hz at which $St \approx 0.4$ (a) and $f = 6$ Hz at which $St \approx 1$ (b), respectively. It is found that for $St \approx 0.4$, 877Phys. Fluids, Vol. 15, No. 4, April 2003 Settling velocity of heavy particles Downloaded 13 Aug 2004 to 128.200.11.160. Redistribution subject to AIP license or copyright, see http://pof.aip.org/pof/copyright.jsp

FIG. 10. (Color) The VGT fluid vertical velocity signals and their corresponding wavelet transform at two different grid frequencies: $f = 3$ Hz (a) and $f = 6$ Hz (b), respectively.
\( \tau_{c1} \approx 0.27 \, s < \tau_{l} \approx 0.42 \, s \) and \( \tau_{c2} \approx 0.15 \, s < \tau_{\lambda} \approx 0.20 - 0.35 \, s \), while \( \tau_{c1} \approx \tau_{l} \approx 0.21 \, s \) and \( \tau_{c2} \approx \tau_{\lambda} \approx 0.12 \, s \) for \( St \approx 1 \). For the case of \( St < 1 \), there is a phase difference between the fluid velocity and the particle motion, because \( \tau_{c1} \) and \( \tau_{c2} \) are much smaller than \( \tau_{l} \) and \( \tau_{\lambda} \), respectively, which in turn gives a larger slip velocity between the fluid and these particles. On the other hand, the particle motion is at the same velocity phase as the fluid turbulent motion when \( St = 1 \), since \( \tau_{c1} \approx \tau_{l} \) and \( \tau_{c2} \approx \tau_{\lambda} \) at which the slip velocity between the fluid and these particles is smallest. The smaller the slip velocity, the smaller the local drag experienced by these heavy particles and thus the higher the particle settling rate. This may explain why the settling rate is a maximum around \( St = 1 \). Furthermore, the cutoff scale may be the Taylor microscale, as found in Figs. 10 and 11. This finding is consistent with that in Yang and Lei\(^7\) that the large eddy simulation can be used to study the particle settling problem because the cut-off scale is an order of magnitude larger than the Kolmogorov scale. However, \( \tau_{l} \) is still an appropriate time scale, since the time scale for the vorticity associated with charac-
IV. CONCLUSION

We measured the settling rate of solid heavy particles in an aqueous stationary near-isotropic turbulence that was generated in the core region between two vertically oscillated grids in a water tank. Initially, a cloud of heavy particles was uniformly released just beneath the water surface. These descending particles then entered the region of interest and interacted with the stationary near-isotropic turbulence. With the appropriate choice of the particle diameters and the grid mesh size, oscillating frequency and stroke, the interaction between particles and turbulence can be investigated for values of the Stokes number ranging from 0 to 2. Using particle tracking and particle image velocimetry, the average set-rioles of the Stokes number ranging from 0 to 2. Using particle mesh size, oscillating frequency and stroke, the interaction acted with the stationary near-isotropic turbulence. With descending particles then entered the region of interest and in-grids in a water tank. Initially, a cloud of heavy particles was erated in the core region between two vertically oscillated an aqueous stationary near-isotropic turbulence that was gen-

This study contributes to measurements of the settling rate of heavy particles in stationary near-isotropic turbulence that may address the discrepancy among previous numerical studies. These results may be relevant to sediment grains in rivers and aerosol particles in the atmosphere. We are currently investigating the case of particle–air interactions in a large cruciform apparatus which consists of two cylindrical vessels. The long vertical vessel provides a uniform distribution of heavy descending particles with their terminal velocity, while the large horizontal vessel is equipped with a pair of counter-rotating fans and perforated plates at each end to generate stationary near-isotropic turbulence. This cruciform apparatus can be used to study the particle settling at reduced, normal, and elevated pressures. Thus, the aerodynamic drag force during the interaction of heavy particles and air turbulence as well as turbulence modification due to the two-way coupling can be experimentally scrutinized for the first time. These results will be published elsewhere in the near future.

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