The Reliability of Mechanical System with Monitor And Improvement of Repair Techniques K. S. Wang

**Definition:**

(1) The reliability concerns with a certain function of mechanical system evaluated by a performance index, once it is below the index, the system stops running.

Here it should be noticed the difference between the function failure and failure mechanisms. In general behind the function failure it may involve several failure mechanisms.

(2) When the system fails, repair should be adopted and then recovery of the function is achieved.

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Assumptions:

A. for the mechanical system:
   The performance decays with memory, e.g. the displacement amplitude which becomes large with time gradually due to the loosing of joint rigidity.

B. for the monitoring system:
   The monitor should be aligned with the mechanical system, the supporting structure also suffers cumulative-type failure mechanism. (i.e. failure with memory)

C. for the repairing capability:
   The repairing capability involves the technique in dealing with different failure mechanisms related to the decay of system performance. Thus it should be learned and enhanced by mistakes.
The primary model:

From definition the reliability of mechanical system with monitoring and repairs if necessary, \( R \), follows

\[
R = R_1 + (R_1 + I)(1 - R_1)R_2 \quad (1)
\]

for performance watch

where

\( R_1 \) : the reliability of mechanical system.
\( R_2 \) : the reliability of monitor system.
\( I \) : the improvement factor about repairing capability.

The second term is added due to the recovery from repair with the monitor reliability \( R_2 \). Once the system needs repair, the mechanical system reliability is improved to \( R_1 + I \).
From assumption A:

\[
\frac{dR_1}{dt} = -(e_1 + c_1 (1 - R_1)^{n_1}) R_1
\]  
(2)

From assumption B:

\[
\frac{dR_2}{dt} = -(e_2 + c_2 (1 - R_2)^{n_2}) R_2
\]  
(3)

From assumption C:

The improvement factor, \( I \), accumulates previous experience in repairs, i.e.

\[
\frac{dI}{dt} = f(R_1) R_2
\]  
(4)

From learning characteristics, \( f(R_1) \) may be assumed as a combination of power law of \( R_1 \) and the failure occurrence, \( 1 - R_1 \), i.e.

\[
f(R_1) = i R_1^m (1 - R_1)
\]  
(5)
Then
\[
\frac{dI}{dt} = iR^m_1 (1 - R_1) R_2
\]  
(6)

Equations (1) (2) and (6) complete the model with initial conditions

\[
R_1 (0) = 1 \quad R_2 (0) = 1 \quad \text{and} \quad I(0) = 0
\]

It should be noted that the system reliability with monitor and repairs is always less than unity, i.e. Eq.(1) follows the constraint

\[
R_1 + I \leq 1
\]

The coefficient \(i\) in Eq.(6) relates to the repairing technique and the power m to the complexity of failure mechanisms involved in the performance evaluation.
Reliability of Cumulative Damage:

Typical examples come from fatigue, surface wear, strength degradation due to corrosion etc. The dynamic reliability can be either expressed by Weibull distributions or followed in the equation.

for multi-failure mechanisms

\[
\frac{dR}{dt} = \begin{cases} 
-\left[e + c(1-R)^n\right]R & \text{.........(7a)} \\
-\left[c_1(1-R)^{n_1} + c_2(1-R)^{n_2}\right]R & \text{.........(7b)} 
\end{cases}
\]

for single failure mechanism

\[
\frac{dR}{dt} = -c^* (1 - R)^{n^*} R^{m^*} \text{ ........... (8)}
\]

the initial condition for (7a) \( R(0) = 1 \)

(7b) \( R(t_0) = R_0 \)

(8) \( R(t_0) = R_0 \)
Example:


The parameters are $e = 10^{-10}$, $c = 1.24 \times 10^{-4}$, $n = 0.88$ for ECN model and $c_1 = 1.2 \times 10^{-4}$, $n_1 = 0.84$ and $c_2 = 1.0 \times 10^{-6}$, $n_2 = 1.5$ for CN12 model and $c^* = 1.0 \times 10^{-4}$, $m^* = 0.88$, $n^* = 0.78$ for CMN model.
Eq(1) relates to the reliability defined by function performance.

However as the failure mechanism is identified, in this case, Eq(1) is modified as

\[ R = R_1 + R_1 (1 + I)(1 - R_1)R_2 \]  \hspace{1cm} (9)

\[
\begin{align*}
    e_1 &= 10^{-3} \\
    e_2 &= 5 \times 10^{-4} \\
    i &= 1.6 \times 10^{-1} \\
    c_1 &= 10^{-1} \\
    c_2 &= 10^{-1} \quad \text{and} \quad n_1 = 1 \quad \text{and} \quad n_2 = 1.5 \\
    m &= 4
\end{align*}
\]