Implementation of edge-preserving regularization for frequency-domain diffuse optical tomography

Liang-Yu Chen, 1 Min-Chun Pan, 1,2 and Min-Cheng Pan 3,*

1Department of Mechanical Engineering, National Central University, Taoyuan County 320, Taiwan
2Graduate Institute of Biomedical Engineering, National Central University, Taoyuan County 320, Taiwan
3Department of Electronic Engineering, Tung-Nan University, Taipei County 222, Taiwan

*Corresponding author: m2pan@mail.tnu.edu.tw

Received 13 April 2011; revised 29 June 2011; accepted 16 September 2011; posted 19 September 2011 (Doc. ID 143760); published 22 December 2011

In this study, we first propose the use of edge-preserving regularization in optimizing an ill-conditioned problem in the reconstruction procedure for diffuse optical tomography to prevent unwanted edge smoothing, which usually degrades the attributes of images for distinguishing tumors from background tissues when using Tikhonov regularization. In the edge-preserving regularization method presented here, a potential function with edge-preserving properties is introduced as a regularized term in an objective function. With the minimization of this proposed objective function, an iterative method to solve this optimization problem is presented in which half-quadratic regularization is introduced to simplify the minimization task. Both numerical and experimental data are employed to justify the proposed technique. The reconstruction results indicate that edge-preserving regularization provides a superior performance over Tikhonov regularization. © 2011 Optical Society of America

OCIS codes: 100.3190, 170.3010, 170.6960.

1. Introduction

Over the past two decades, interest in the use of near-infrared (NIR) diffuse optical tomography (DOT) [1–3] has increased. Typically, NIR DOT is a noninvasive and nonradioactive functional imaging modality, thereby estimating the NIR absorbing and scattering properties of the tissue. Its clinical application of imaging breast cancer is beginning to demonstrate the feasibility of extracting physiologically relevant information of optical-property images reconstructed from diffuse optical measurements. As known, NIR DOT, however, produces low-resolved images due to the highly scattered photon fields, which limits its further clinical application. Usually, an NIR DOT system mainly concerns two issues, a forward measurement and an inverse solution primarily relating to a measuring device and numerical computation software, respectively. Thus, the drawback of low resolution can be remedied by either a novel hardware design such as using multimodality, the optimization of the geometry for arrangement of optical fibers, or an innovative approach of computation such as introducing prior information, varying parameters, filtering techniques, or constraints in the image reconstruction.

The system using a number of wavelengths was developed to reflect a better-quality performance such as a low cross talk or a high separability [4–7]. Additionally, the wavelength optimization [8–10] to find optimal spectral ranges was examined, and the choice of the laser wavelengths has a strong impact on improving accuracy of optical properties. With multispectral NIR imaging, it usually yields evident separation of scattering from absorption.

Furthermore, multimodality systems have been developed for biomedical application or clinic diagnosis [11–17]. Some studies coregistered x-ray and optical imaging systems to result in a better understanding between the structural contrast and the functional contrast [11, 12], integrated an ultrasound...
Following the above introduction, this paper is organized as follows. In Section 2 we briefly review TR to find the inverse solution from the diffusion equation for NIR DOT in the FD; in addition, the approach to incorporating the edge-preserving function into regularization is discussed in detail. Subsequently, both numerical simulations and reconstructions using experimental measurements are demonstrated in Section 3, where discussion and evaluation are provided as well. Finally, some remarks are concluded in Section 4.

2. Forward and Inverse Models

The goal of DOT is to estimate the distribution of the optical properties (the absorption coefficient and the diffusion coefficient) in tissue from one-dimensional (1D) boundary measurements; it is called an imaging process or an inverse problem. For the purpose of determining the optical-properties images, a forward model is needed to describe the physical relation between the boundary measurements of tissue and the optical properties that characterize the tissue.

In the imaging process of NIR DOT, the reconstruction of optical-property images can be done iteratively using a Newton method, requiring inversion of a matrix; however, such a matrix cannot be directly inverted due to resulting in an ill-conditioned problem; hence, the method of regularization is utilized to resolve such a problem. Basically, the solution to search for an optimal reconstruction is based on an initial estimate of property distribution, and then the difference between the measured and the calculated photon densities is minimized by iteratively updating the set of absorption and scattering/diffusion coefficients.

To express the above in theory, a forward problem in DOT is described in Subsection 2.A, and then the use of the regularization method in an inverse problem is explained in Subsection 2.B, where EPR is proposed. Following that, the approach implemented in the numerical way is described in Subsection 2.C.

A. Forward Problem in DOT

In general, such a forward model that gives the description of this physical relation is the diffusion equation,

$$\nabla \cdot \kappa(r) \nabla \Phi(r, \omega) - \frac{\mu_a(r) - i \omega}{c} \Phi(r, \omega) = - S(r, \omega).$$

(1)

where $\Phi(r, \omega)$ is the photon density at position $r$, $\omega$ is the light modulation frequency, $S(r, \omega)$ is the isotropic source term, $c$ is the speed of light in tissue, and $\mu_a$ and $\kappa$ denote the optical absorption and diffusion coefficients, respectively. In addition, the finite-element method (FEM) is applied on Eq. (1) with a Robin (type-III) [10,13] boundary condition to solve this forward problem, i.e., calculating the photon density for a given set of optical property within the tissue.
B. Inverse Problem in DOT

Owing to the nonlinearity with respect to the optical properties, an analytic solution to the inverse problem in DOT is absent. Instead, the numerical way of obtaining the inverse solution is to iteratively minimize the difference between the measured diffusion photon density data, $\Phi^M$, around the tissue and the calculated model data, $\Phi^C$, from solving the forward problem with the current estimated optical properties. This data–model misfit difference is typically defined as follows:

$$\chi^2 = \sum_{i=1}^{N_M} |\Phi^C_i - \Phi^M_i|^2,$$

(2)

where $N_M$ is the number of measurements.

By means of the first-order Taylor series to expand $\Phi$, one can get Eq. (3),

$$(\Phi^M) \approx (\Phi^C) + \left[ \frac{\partial \Phi^C}{\partial \mu_a} \right] (\Delta \mu_a) + \left[ \frac{\partial \Phi^C}{\partial \kappa} \right] (\Delta \kappa),$$

(3)

where the goal is to reach $\Phi^M$ from the current $\Phi^C$. As well, the vector $(\Delta \mu_a)$ and $(\Delta \kappa)$ denote the updates for $\mu_a$ and $\kappa$, respectively, with dimension $N_N$ (the number of total nodes in the finite-element mesh), and thus the dimension of the matrices $[\partial \Phi^C / \partial \mu_a]$ or $[\partial \Phi^C / \partial \kappa]$ is $N_M \times N_N$. From Eq. (2), the inverse problem in DOT can be formulated as

$$\left[ \frac{\partial \Phi^C}{\partial \mu_a} \quad \frac{\partial \Phi^C}{\partial \kappa} \right] \left( \Delta \mu_a \quad \Delta \kappa \right) = (\Phi^M - \Phi^C),$$

(4)

or simply denoted as $J \Delta \chi = \Delta \Phi$, where $J = [\partial \Phi^C / \partial \mu_a, \partial \Phi^C / \partial \kappa]$ is the Jacobian matrix, i.e., the rate of change of model data with respect to optical parameters, and $\Delta \chi$ means $\left( \frac{\Delta \mu_a}{\Delta \kappa} \right)$. It is worth emphasizing that Eq. (4) can be efficiently calculated using the adjoint method [34].

However, solving this linearized inverse problem from Eq. (4) usually runs into difficulty with an ill-conditioned problem, which typically happens as the number of model parameters increases, so as to solve the inverse problem by means of regularization to remedy such a drawback. In Subsections 2.B.1 and 2.B.2, the regularization of an inverse problem and the corresponding algorithms for computing a regularized solution are described in detail.

1. Tikhonov Regularization

In TR, the inverse problem in DOT is formulated as an optimization of the damped least-squares problem,

$$\min_{\Delta \chi} \{ Q_{Tk}(\Delta \chi) \} = \min_{\Delta \chi} \{ \| J \Delta \chi - \Delta \Phi \|_2^2 + \lambda^2 \| L \Delta \chi \|_2^2 \},$$

(5)

where $L$ is the dimensionless regularization matrix and $\lambda$ is the regularization parameter. Specifically, Eq. (5) is called a zero-order TR, provided that the identity matrix is adopted as regularization matrix, i.e., $L = I$.

In order to apply Newton’s method to minimize this proposed objective function, one can rewrite it as

$$Q_{Tk}(\Delta \chi) = \| J \Delta \chi - \Delta \Phi \|_2^2.$$

(6)

By using Newton’s method for minimizing $Q_{Tk}(\Delta \chi)$, a necessarily first-order condition is that $\partial Q_{Tk}(\Delta \chi) / \partial \Delta \chi = 0$, leading to

$$\frac{\partial Q_{Tk}(\Delta \chi)}{\partial \Delta \chi} = [J^T \lambda L^T] \left[ \frac{J \Delta \chi - \Delta \Phi}{\lambda L \Delta \chi} \right] = (J^T J + \lambda^2 L^T L) \Delta \chi - J^T \Delta \Phi = 0,$$

(7)

where the superscript $T$ represents the matrix transpose operator. Thus, the update equation is obtained from Eq. (7), i.e.,

$$(J^T J + \lambda^2 L^T L) \Delta \chi = J^T \Delta \Phi,$$

(8)

which is solved iteratively by starting from the initial guess of optical properties and ending up with the stopping criterion being reached.

2. Edge-Preserving Regularization

In EPR [31–33], the inverse problem in DOT is also considered a damped least-squares problem in which the objective function is defined as

$$Q_{Ep}(\Delta \chi) = \| J \Delta \chi - \Delta \Phi \|_2^2 + \lambda^2 \sum_k \sum_l \phi[(D_l \Delta \chi)_k],$$

(9)

where the index $k$ is in the lexicographical order, the index $l$ may represent the horizontal, vertical, or diagonal direction, and $\lambda$, again, is the regularization parameter that balances the first term (a residual term) and the second term (a regularization or a priori term).

The $\phi$ in the a priori term of the objective function determines the regularization imposed on every value of the first-order difference $D_l \Delta \chi$, which is used to detect the discontinuities of the update vector $\Delta \chi$ in specific direction $l$. In order to preserve edges, $\phi$ is also required to satisfy three conditions [33] as follows:

- $\frac{\phi(t)}{\lambda t}$ is continuous and strictly decreasing on $[0, +\infty)$.
- $\lim_{t \to +\infty} \frac{\phi(t)}{\lambda t} = 0$.
- $\lim_{t \to 0+} \frac{\phi(t)}{\lambda t} = M$, $0 < M < +\infty$.

Suppose that $Q_{Ep}(\Delta \chi)$ has a minimum over $\Delta \chi$; then the derivative of $Q_{Ep}(\Delta \chi)$ with respect to $\Delta \chi$ is zero; hence, Eq. (9) leads to
\[ J^T J \Delta \chi - J^T \Delta \Phi - \lambda^2 \Delta_{\text{weighted}} \Delta \chi = 0 \]  
\[ \Delta \chi_{n+1} = \arg \min_{\Delta \chi} \{ Q_{\text{Ep}}(\Delta \chi, b^{n+1}) \} \]

where \( \Delta \chi_{n+1} \) is the new update vector, \( Q_{\text{Ep}}(\Delta \chi, b^{n+1}) \) is the proposed objective function, and \( \lambda^2 \Delta_{\text{weighted}} \Delta \chi \) is the matrix dependent upon \( \Delta \chi \) (Appendix A of [33]); i.e., the matrix–vector multiplication \( \Delta \chi = \text{equivalent to a nonstationary filtering of } \Delta \chi \). Therefore, the minimization of the proposed objective function is not an easy task because of nonlinearity shown in Eq. (9).

Introduced to simplify the minimization task is half-quadratic regularization, of which the principle is to bring in the auxiliary variable \( b = (b_1, b_2, ..., b_l) \), capable of making the first-order necessary condition linear in \( \Delta \chi \) and then making the manipulation of the necessary condition easier. After applying half-quadratic regularization, it is proven that the original objective function \( Q_{\text{Ep}}(\Delta \chi) \) can be written as the minimum of a dual energy [33], i.e.,

\[ Q_{\text{Ep}}(\Delta \chi) = \inf_{b} Q_{\text{Ep}}^c(\Delta \chi, b), \]

where

\[ Q_{\text{Ep}}^c(\Delta \chi, b) = \| J \Delta \chi - \Delta \Phi \|^2_2 + \lambda^2 \sum_{t} (\{ b_t \} (D_t \Delta \chi^2)_k + \varphi_t (b_t) k) \].

(12)

Note that Eq. (12) has the property that this dual energy is quadratic in \( \Delta \chi \) when \( b \) is fixed. As a result, the first-order necessary condition is linear in \( \Delta \chi \) when the auxiliary variable is fixed.

Adopted for the reconstruction of the optical-property images by iteratively updating \( \Delta \chi \), a strategy is to calculate the update vector \( \Delta \chi \) based on minimization of this dual energy. First, \( \Delta \chi^a \) is fixed at iteration step \( n + 1 \), and \( b^{n+1} \) is simply computed using the expression obtained from minimization of \( Q_{\text{Ep}}^c(\Delta \chi^a, b) \) (Appendix B of [33]) such that

\[ \{ b_t^{n+1} \} = \arg \min_{\{ b_t \}} Q_{\text{Ep}}^c(\Delta \chi^a, \{ b_t \}). \]

(13)

Then the new update vector \( \Delta \chi_{n+1} \) is the solution of the update equation obtained from minimization of \( Q_{\text{Ep}}(\Delta \chi, b^{n+1}) \) such that

\[ \Delta \chi_{n+1} = \arg \min_{\Delta \chi} \{ Q_{\text{Ep}}(\Delta \chi, b^{n+1}) \} \]

\[ = \min \{ J^T J + \lambda^2 \Delta_{\text{weighted}} \Delta \chi \} \]

(14)

where \( \Delta \chi_{n+1} = \sum_t D_t b_t^{n+1} D_t^T \) and \( b_t^{n+1} = \text{diag}(b_t^{n+1}). \)

The update equation, Eq. (14), is solved iteratively from the current estimated optical properties until the stopping criteria are met. In the study, one can use a generalized Lorentzian function as the edge-preserving weighting function, i.e.,

\[ q^e(t) = \frac{(\gamma^2 t)^m}{(\gamma^2 t^2 + \gamma^2)^m} \]

(15)

where \( \gamma \) and \( m \) are adjustable parameters to change the behavior of the edge-preserving weighting function. The use of such a generalized Lorentzian function is for its adaptive characteristics.

C. Implementation in Image Reconstruction for DOT

In this study, a model-based optical-property reconstruction is employed. In such a model-based reconstruction algorithm, an optical-property image of tissue is reconstructed first by comparing the measured diffusion photon density data and the theoretical prediction based on the diffusion equation. Then the inverse solution is obtained through a least-squares minimization problem in which the optimization strategy is adopted with TR or EPR. The entire flowchart is indicated in Fig. 1, where there is no need to alter too much the computer code of the FEM-based reconstruction for TR when EPR is used.

When implementing image reconstruction with EPR for DOT, there is involved an extra configuration process in calculating the first-order difference of the current node and the neighboring node in a specific direction, \( D_t \Delta \chi \), shown in the regularization term of Eqs. (13) and (14). Ideally, the first-order difference in the horizontal direction is defined as \( (D_t f)_{i,j} = f_{i,j+1} - f_{i,j} \)/\( \delta \) for a discrete two-dimensional (2D)
image $f_{i,j}$, where $\delta$ is a scaling parameter that tunes the value of the first-order difference. By contrast, the matrix $D_0$ of FEM-based image reconstruction, thus, can be built by the similar concept in order to apply to the FEM mesh, i.e.,

$$
[D_0]_{i,j} = 
\begin{cases} 
-1/\delta_{i,j}, & i = j \\
1/\delta_{i,j}, & i \neq j \text{ and } i,j \text{ are neighbors in } l \text{ direction}, \\
0, & i \neq j \text{ and } i,j \text{ are not neighbors}
\end{cases}
$$

where $\delta_{ij}$ is the distance between the node $i$ and the node $j$. Since triangular elements are considered in the 2D finite-element forward solver of image reconstruction [Fig. 2(b)], six triangular finite elements can be regarded as a hexagon; i.e., there are always six neighboring nodes around an interior node and three different directions are calculated here for the first-order difference as depicted in Fig. 2(c).

3. Results and Discussion

In this section, the proposed reconstruction algorithm with EPR strategy is evaluated through synthetic images of optical property and measurements of experimental phantoms; the former have four simulation cases and the latter have two experimental examples. For the purpose of comparison, reconstructions with the use of TR are also presented. In the simulation, image reconstructions were conducted under idealized conditions with no measurement noise in the simulated measured data. For simulation cases, the synthetic images were designed as a homogeneous background with different locations, sizes, or contrast levels of heterogeneity as well as a more complex distribution (layered domain) of optical property with an embedded inclusion, where both the absorption and the reduced scattering coefficients of inclusion were increased by the specific times relative to the background. Here it is noticed that results are displayed in terms of the reduced scattering coefficient ($\mu_s'$), which actually is inversely proportional to the diffusion coefficient ($\kappa$) usually adopted as a reconstruction parameter in an image formation algorithm. In the experimental work, the evaluation of the image reconstruction algorithm with EPR is presented as well under conditions of noisy measurement data acquired in laboratory environments.

The test geometry is a phantom with 80 mm in diameter, of which optical properties, $\mu_s' = 0.6 \text{ mm}^{-1}$ and $\mu_a = 0.006 \text{ mm}^{-1}$, were used for the background medium. The modulation frequency was selected to be 20 MHz, and multiple excitation and measurement positions were used to extract (for simulations) or collect (for experiments) boundary information used in the reconstructions. Specifically, for all simulations or experiments, it was assumed that 16 measurement locations, equally spaced around the circular circumference, for each of 16 excitation positions, were acquired, which yielded a total of 256 amplitude and 256 phase-shift observations for each image reconstruction. Meanwhile, the finite-element forward solution with the Robin (type-III) boundary condition was obtained, and the finite-element mesh consisting of 4225 nodes and 8192 triangle elements was used to generate simulated data [Fig. 2(a)]. A second mesh consisting of 817 nodes and 1536 triangle elements was generated and used in the image reconstruction procedure [Fig. 2(b)]. All reconstructed images reported in this study started from the optical properties of the homogeneous background except for the layered-domain simulation case; for all cases, 30 iterations were used during the reconstruction procedure, and the stopping criterion, $||\Phi^{n-1} - \Phi^n||^2 / ||\Phi^n||^2 < 10^{-3}$, was employed.

As mentioned above, a generalized Lorentzian function was used as the edge-preserving weighting function in this study even though different weighting functions could be employed in EPR. Since the edge-preserving weighting function is required to satisfy the condition that it is strictly decreasing on $[0, +\infty)$, it has implicitly made the assumption that a large value of the gradient corresponds to an edge and preserves such an edge by assigning a small weight value to a large gradient value. The edge-preserving weighting function with slowly decreasing rate would result in reconstructing blur images, while the weighting function with rapidly decreasing rate would produce sharp images but also make the reconstruction procedure unstable. Therefore, one can be more flexible to change the behavior of the edge-preserving weighting function through the adjustable parameter $\gamma$ and $m$ in the proposed generalized Lorentzian function. The values of $\gamma = 0.0025$, a comparable order of magnitude with that of the gradient value generated during reconstruction procedure, and $m = 1$ for the use of the proposed edge-preserving weighting function show moderate decreasing rate and were adopted in this study. Moreover, the regularization parameter $\lambda$ presented in the regularization methods controls the ratio of the regularization term relative to the residual term. A large $\lambda$ favors a small norm of the regularization term at the cost of making distinguishing tumors/inclusions from background difficult, whereas a small $\lambda$ would lead to the reconstructed images not being sufficiently constrained.
images dominated by oscillations. For the cases presented here, the regularization parameter $\lambda = \max[\text{diag}(J^TJ)]$ appears to provide excellent reconstructed optical-property images from both simulated and experimental data.

For all cases illustrated below, the first and the second rows of the figure represent the corresponding images or 1D profiles of $\mu_a$ and $\mu_s$, respectively, and the left, the middle, and the right column images display the exact images, the reconstructed images using TR, and the reconstructed images using EPR, respectively. To provide a more detailed assessment of these images, 1D profiles are essential for comparison, where the left and the right columns represent the reconstruction using TR and EPR, respectively.

A. Reconstructions from Simulated Data

In each simulation, the measured data used in the reconstruction of these cases were obtained by assuming that it is the forward solution from the

![Fig. 3](image3.png)

Fig. 3. (Color online) Simulated reconstructions of both the absorption and the reduced scattering images with a 3:1 contrast level for inclusions having the same size but at different off-center distances. (a) Exact absorption image, (b) reconstruction using TR, and (c) reconstruction using EPR; (d) exact reduced scattering image, (e) reconstruction using TR, and (f) reconstruction using EPR.

![Fig. 4](image4.png)

Fig. 4. (Color online) Simulated reconstructions of both the absorption and the reduced scattering images with a 3:1 contrast level for inclusions having different geometric sizes at a same off-center distance (20 mm). (a) Exact absorption image, (b) reconstruction using TR, and (c) reconstruction using EPR; (d) exact reduced scattering image, (e) reconstruction using TR, and (f) reconstruction using EPR.

![Fig. 5](image5.png)

Fig. 5. (Color online) Comparison between exact (dotted line) and simulated (solid line) reconstructions by 1D circular profiles with a radius of 20 mm for the images in Fig. 4. (a), (b) 1D profiles of $\mu_a$ using TR and EPR, respectively; (c), (d) 1D profiles of $\mu_s$ using TR and EPR, respectively.
finite-element diffusion model with the exact optical properties in place. Figure 3 shows a case of the phantom image having three same size inclusions with a 3:1 contrast level with respect to the background, where the distances between the centers of each inclusion and the background are 15, 20, and 25 mm for those at the bottom right, the top, and the bottom left, respectively, as shown in Figs. 3(a) and 3(d). As can be seen, the contrast was enhanced much more by the use of EPR [Figs. 3(c) and 3(f)] than the reconstructed images using TR [Figs. 3(b) and 3(e)].

Figure 4 presents image reconstructions for inclusions with different geometric sizes, which are 6, 10, and 14 mm in diameter at the bottom right, the top, and the bottom left, respectively, and all of three inclusions are located at an off-center distance 20 mm. Apparently, Fig. 4 demonstrates a considerable contrast improvement in the reconstructed images.
when EPR was invoked, especially for the smallest inclusion at the bottom right region of the phantom [Fig. 4(d)]. Corresponding to Fig. 4, Fig. 5 shows 1D profiles in which the optical-property distributions are depicted along a circular profile of a radius of 20 mm, where higher contrast was reconstructed using EPR [Fig. 5(d)] but even overestimated.

Figure 6 displays image reconstructions for a phantom with three absorbing and/or scattering inclusions embedded in a homogeneous background. This case is specified with the optical-property values of $\mu'_a = 0.6 \text{ mm}^{-1}$ and $\mu_a = 0.018 \text{ mm}^{-1}$ for the bottom right inclusion, $\mu'_a = 1.2 \text{ mm}^{-1}$ and $\mu_a = 0.012 \text{ mm}^{-1}$ for the top one, and $\mu'_s = 0.9 \text{ mm}^{-1}$ and $\mu_s = 0.012 \text{ mm}^{-1}$ for the smallest inclusion at the bottom right region of the phantom [Fig. 4(d)]. Corresponding to Fig. 4, Fig. 5 shows 1D profiles in which the optical-property distributions are depicted along a circular profile of a radius of 20 mm, where higher contrast was reconstructed using EPR [Fig. 5(d)] but even overestimated.

<table>
<thead>
<tr>
<th>Table 1. Evaluation on Contrast and Size Resolutions of 1D Profiles for TR and EPR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_a$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Fig. 5</td>
</tr>
<tr>
<td>Fig. 7</td>
</tr>
<tr>
<td>Fig. 9</td>
</tr>
<tr>
<td>Fig. 11</td>
</tr>
<tr>
<td>Fig. 13</td>
</tr>
<tr>
<td>$\mu'_a$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Fig. 5</td>
</tr>
<tr>
<td>Fig. 7</td>
</tr>
<tr>
<td>Fig. 9</td>
</tr>
<tr>
<td>Fig. 11</td>
</tr>
<tr>
<td>Fig. 13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2. Evaluation on Contrast and Size Resolutions of 2D Images for TR and EPR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_a$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Fig. 3</td>
</tr>
<tr>
<td>Fig. 4</td>
</tr>
<tr>
<td>Fig. 9</td>
</tr>
<tr>
<td>Fig. 11</td>
</tr>
<tr>
<td>Fig. 10</td>
</tr>
<tr>
<td>Fig. 12</td>
</tr>
<tr>
<td>$\mu'_a$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Fig. 3</td>
</tr>
<tr>
<td>Fig. 4</td>
</tr>
<tr>
<td>Fig. 9</td>
</tr>
<tr>
<td>Fig. 11</td>
</tr>
<tr>
<td>Fig. 10</td>
</tr>
<tr>
<td>Fig. 12</td>
</tr>
</tbody>
</table>
$\mu_a = 0.006 \text{ mm}^{-1}$ for the bottom left one. Without doubt, optical-property images were reconstructed in different levels either using TR or EPR; however, a little cross talk is existing between absorption and scatter features. Likewise, Fig. 7 depicts 1D circular profiles of the images shown in Fig. 6, where sharper edges and higher contrast were reconstructed when the reconstruction involved EPR, especially for the image of the reduced scattering coefficient [Fig. 7(d)].

A two-layered phantom with an inclusion embedded inside the inner layer was used to evaluate the ability of EPR to resolve a more realistic structure in the simulation of breast imaging. The outer layer has optical-property values of $\mu_a = 0.006 \text{ mm}^{-1}$ and $\mu'_s = 0.6 \text{ mm}^{-1}$ and the inclusion with $\mu_a = 0.024 \text{ mm}^{-1}$ and $\mu'_s = 0.6 \text{ mm}^{-1}$ was placed inside the inner layer having the optical-property values of $\mu_a = 0.012 \text{ mm}^{-1}$ and $\mu'_s = 1.2 \text{ mm}^{-1}$. Reconstructed images for this two-layered domain case are shown in Fig. 8, in which iteration started from optical properties of the inner layer. Compared with those using the TR algorithm, shown in Figs. 8(b) and 8(e), the reconstructed images were highly enhanced using the proposed edge-preserving reconstruction algorithm, shown in Figs. 8(c) and 8(f), where two layers and the inclusion are identified without interaction. For a detailed comparison of the reconstructions, 1D profiles of the images (Fig. 8) were extracted along the horizontal line through the centers of both the inclusion and the background, shown as Fig. 9. Obviously, our proposed EPR method has performance superior to TR, where the inclusion was reconstructed very clearly, comparing Figs. 9(b) and 9(d) with Figs. 9(a) and 9(c).

To provide a quantitative assessment for these reconstructed images with using TR or EPR, two measures (contrast resolution and size resolution) defined in [30] were determined over the region of interest (ROI). The contrast resolution is defined to evaluate the resolution on the contrast of optical-property values of the inclusions relative to that of

![Fig. 10. (Color online) Reconstructed images from experimental data obtained from an eccentrically located inclusion having a 4:1 contrast level with respect to the background medium. (a) Exact absorption image, (b) reconstruction using TR, and (c) reconstruction using EPR; (d) exact reduced scattering image, (e) reconstruction using TR, and (f) reconstruction using EPR.](image)

![Fig. 11. (Color online) Comparison between exact (dotted line) and simulated (solid line) reconstructions by 1D circular profiles with a radius of 12.5 mm for the images in Fig. 10. (a), (b) are 1D profiles of $\mu_a$ using TR and EPR, respectively; (c), (d) are 1D profiles of $\mu'_s$ using TR and EPR, respectively.](image)
the background region, and the size resolution is
designed to evaluate the resolution on the size over all
inclusions. The resolution value would be equal to a
unit when the reconstructed result is exactly same
as the expected result. Tables 1 and 2 present the
evaluation results from calculating these two measures
over 1D ROI and 2D ROI for all images shown in
Figs. 3–9, where 1D ROI is chosen as the 1D profile
of the corresponding images and 2D ROI is the whole
reconstructed images. The quantitative measures in
Tables 1 and 2 verify that the reconstructed optical-
property images using EPR result in an obvious
enhancement in terms of the recovery of the inclusion
contrast and size; there is especially considerable
improvement in the recovery of the layered-domain case.

B. Reconstructions from Experimental Data

In this section, it is validated that our proposed reg-
ularization method can still perform effectively even
under conditions of measurement noise and model
mismatch between the measured data and the for-
ward solution. A cylindrical phantom of a diameter
of 50 mm was made for the following reconstructions
using experimental data [35]. Figure 10 displays the
images reconstructed from experimental data col-
lected from the boundary of a phantom with an ec-
centrically located inclusion (12.5 mm off center
along the horizontal axis at 180°) having a 4:1
contrast level with respect to the background. For
comparison purposes, the exact optical-property
images are also shown in Figs. 10(a) and 10(d). As
demonstrated in Fig. 10, the improved images
resulting from the utilization of EPR, Figs. 10(c)
and 10(f), show superior quality compared with those
using TR, Figs. 10(b) and 10(e). Considering further
quantitative information, Fig. 11 illustrates 1D cir-
cular profiles through the center of the inclusion at
a radius of 12.5 mm; higher contrast was built consid-
erably in Figs. 11(b) and 11(d) when comparing two
regularization methods with each other.

Figure 12 demonstrates the reconstruction of two
eccentrically located inclusions having a 4:1 contrast

Fig. 12. (Color online) Reconstructed images from experimental
data obtained from two eccentrically located inclusions having a
4:1 contrast level with respect to the background medium.
(a) Exact absorption image, (b) reconstruction using TR, and
(c) reconstruction using EPR; (d) exact reduced scattering image,
(e) reconstruction using TR, and (f) reconstruction using EPR.

Fig. 13. (Color online) Comparison between exact (dotted line) and simulated (solid line) reconstructions by 1D circular profiles through
the centers of two inclusions at a radius of 12.5 mm for the images in Fig. 12. (a), (b) 1D profiles of μ_a using TR and EPR, respectively; (c),
(d) 1D profiles of μ_s using TR and EPR, respectively.
level with respect to the background medium. As expected, the images reconstructed by the application of EPR show that the contrast of the absorption and the reduced scattering coefficient was highly enhanced over the reconstructed images by TR. Also, Fig. 13 provides a more detailed assessment of these images by 1D circular profiles through the centers of both inclusions at a radius of 12.5 mm. Obviously, sharper edges and higher contrast for two inclusions were reconstructed in those reconstructed from EPR, shown in Figs. 13(b) and 13(d).

Further quantitative information about the contrast resolution and the size resolution of the images shown in Figs. 10–13 are provided in Tables 1 and 2. It is clearly demonstrated that the reconstructions for \( \mu_a \) and \( \mu'_s \) can be enhanced by the use of EPR further than TR.

4. Conclusions

In this paper, an image reconstruction method has been proposed for DOT in the FD by introducing EPR, which can be straightforwardly implemented in the FEM-based image reconstruction algorithm. It has been validated with both simulations and experiments that the contrast for the absorption and the reduced scattering coefficients is improved by the use of EPR over that reconstructed by TR. One reason for this contrast enhancement would be that the potential function with edge-preservation conditions was used in the objective function Eq. (9) to preserve this feature and result in the contrast enhancement. Although promising successes with the EPR technique have been presented, the improvement of the proposed method is limited in some situations. Examining the quantitative measures in Tables 1 and 2, it is interesting to note that the reconstructed \( \mu_a \) images show a more dramatic contrast and size improvement than the reconstructed \( \mu'_s \) images, implying that both values of the adjustable parameters \( \gamma \) and \( m \) presented in the edge-preserving weighting function should be employed in difference between the absorption and the reduced scattering coefficients. Furthermore, it is anticipated that the EPR method proposed here can be useful for three-dimensional DOT, although the results displayed in this study have concentrated on 2D image reconstruction.

The authors would like to acknowledge the funding support from the grants by the Veteran General Hospital/University System of Taiwan Joint Research Program (VGHUST97-P3-13) and the National Science Council of Taiwan (NSCT) (NSC 98-2221-E-236-013 and NSC 98-2221-E-008-083-MY3).

References


linear inverse solution with multiple priors in diffuse optical


